

SPARSFOLIO SAMPLE ADMISSION TEST

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FULL SOLUTIONS TO ALL THE FIVE PROBLEMS TO BE EMAILED WITHIN 2 HOURS
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PROBLEM 1

Show that the process X_t is a martingale,

$$X_t = \cosh(\eta B(t)) \exp(-\eta^2 t) + [B(t)]^4$$

where $B(t)$ is the standard Brownian motion, and η is any positive real number.

PROBLEM 2

Given that $K_1 < K_2 < K_3$, a derivative pays

$$\frac{1}{\min(\max(\min(S, K_1), K_2), K_3)}$$

Derive a model independent hedge in terms of a portfolio of vanilla options.

PROBLEM 3

There are 5 yellow, 10 red, 20 blue, 25 green, 30 purple balls, you keep taking them out randomly, what is the probability that when you take the last yellow out, there will be red and green balls, as a sum together more than twice the remaining purple balls and at least there are still two purple balls left.

PROBLEM 4

Given that $B(t)$ is the standard Brownian motion and define $\Pi = \{t_0, t_1, \dots, t_n\}$ be a partition of $[0, T]$, show that

$$\lim_{\Pi \rightarrow 0} \sum_{j=0}^{n-1} B(t_j^*) (B(t_{j+1}) - B(t_j)) = \frac{1}{2} B^2(T)$$

PROBLEM 5

Under any valid space, compute

$$\int_{t=0}^T \sqrt{dt}$$